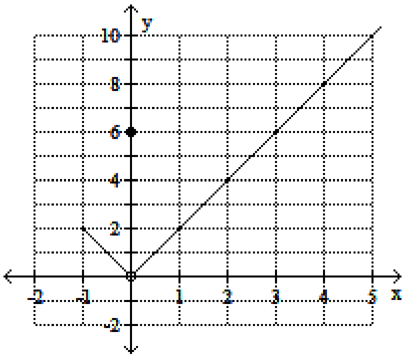
MIDTERM REVIEW

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

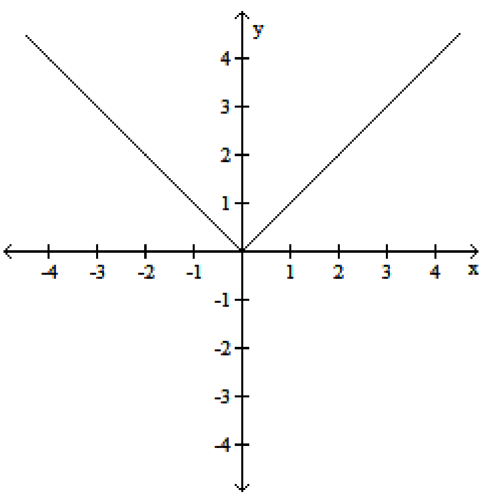
Use the graph to evaluate the limit.

1)



$\lim_{x \to 0} f(x)$

2) $\lim_{x \to 0} f(x)$



Complete the table and use the result to find the indicated limit.

3) If $f(x) = x^2 + 8x - 2$, find $\lim_{x \to 2} f(x)$.

x	1.9	1.99	1.999	2.(
f(x)				

4) If $f(x) = x^2 - 5$, find $\lim_{x \to 0} f(x)$.

x	-0.1	-0.01	-0.001
f(x)			

Find the limit, if it exists.

5) $\lim_{x \to 3} (3x + 4)$

6) $\lim_{x \to 0} (\sqrt{x} - 2)$

7) $\lim_{x \to 4} \sqrt{x^2 + 2x + 1}$

8) $\lim_{x \to \pi/2} \frac{\sin x}{x}$

9) $\lim_{x \to \pi/2} \frac{\cos x}{1 - \pi}$

Find the average rate of change of the function over the given interval.

10) $y = x^2 + 4x$, [2, 4]

11) $y = \frac{3}{x - 2}$, [4, 7]

12) $y = -3x^2 - x$, [5, 6]

Find the limit if it exists.

13) $\lim_{x \to 20} (18 - 5x)$

14) $\lim_{x \to -3} 4x(x + 10)(x - 7)$

Find the limit, if it exists.

15) $\lim_{x \to 0} \frac{x^3 + 12x^2 - 5x}{5x}$

16) $\lim_{x \to 8} \frac{x^2 - 64}{x - 8}$

17) $\lim_{x \to -6} \frac{x^2 + 4x - 12}{x^2 + 3x - 18}$

Evaluate $\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$ for the given x_0 and function f.

18) $f(x) = 4x^2 + 4$ for $x_0 = 4$

19) $f(x) = \frac{x}{4} + 8$ for $x_0 = 8$

Find the limit.

20) $\lim_{x \to 5^+} \sqrt{\frac{7x^2}{6 + x}}$

21) $\lim_{x \to 0.5^-} \sqrt{\frac{x + 1}{x + 2}}$

22) $\lim_{x \to 4^-} \frac{\sqrt{5x}(x - 4)}{|x - 4|}$

Find the limit using $\lim_{x = 0} \frac{\sin x}{x} = 1$.

23) $\lim_{x \to 0} \frac{\sin 5x}{x}$

24) $\lim_{x \to 0} \frac{\sin 5x}{\sin 4x}$

25) $\lim_{x \to 0} \frac{\sin x \cos 4x}{x + x \cos 5x}$

Find the limit, if it exists.

26) $\lim_{x \to \infty} \frac{x^2 - 4x + 11}{x^3 - 8x^2 + 19}$

27) $\lim_{x \to \infty} \frac{3x + 1}{8x - 7}$

28) $\lim_{x \to \infty} \frac{6x^3 - 4x^2 + 3x}{-x^3 - 2x + 5}$

29) $\lim_{x \to -\infty} \frac{6x^3 + 2x^2}{x - 6x^2}$

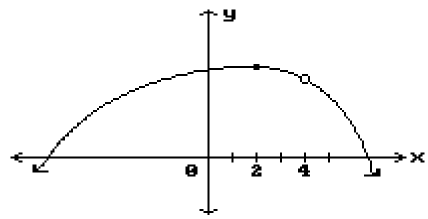
Find the limit.

30) $\lim_{x \to (-2)^+} \frac{1}{x + 2}$

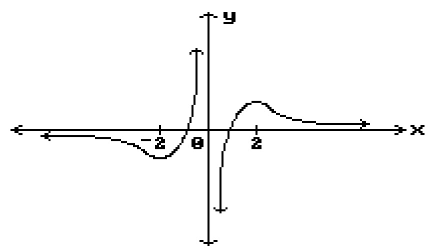
31) $\lim_{x \to (-\pi/2)^-} \sec x$

Find all points where the function is discontinuous.

32)



33)



Find the equation for the tangent to the curve at the given point.

34) $f(x) = x^2 + 5x$; (4, 36)

35) $f(x) = 5x^2 + x$; (-4, 76)

36) $f(x) = 3x^2 + 5x - 7$; (-2, -5)

Calculate the derivative of the function. Then find the value of the derivative as specified.

37) $f(x) = 5x + 9$; $f'(2)$

38) $g(x) = x^3 + 5x$; $g'(1)$

39) $f(x) = \frac{8}{x}$; $f'(-1)$

Find the slope of the tangent line at the given value of the independent variable.

40) $f(x) = 3x + \frac{9}{x}$, $x = 4$

41) $g(x) = \frac{8}{9 + x}$, $x = 3$

Find an equation of the tangent line at the indicated point on the graph of the function.

42) $y = f(x) = \frac{x^3}{2}$, $(x, y) = (8, 256)$

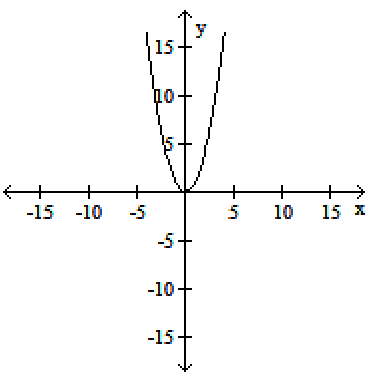
43) $y = f(x) = x^2 - x$, $(x, y) = (-2, 6)$

44) $y = f(x) = 6\sqrt{x} - x + 9$, $(x, y) = (36, 9)$

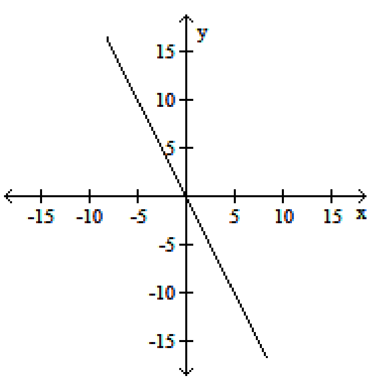
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The graph of a function is given. Choose the answer that represents the graph of its derivative.

45)



A)



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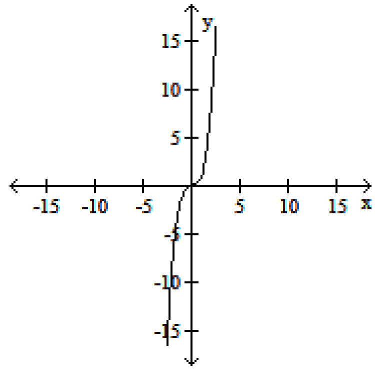

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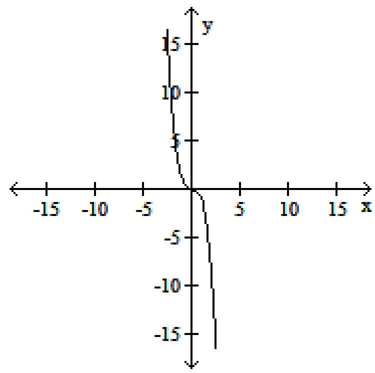

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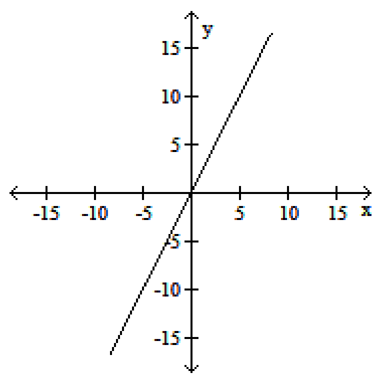

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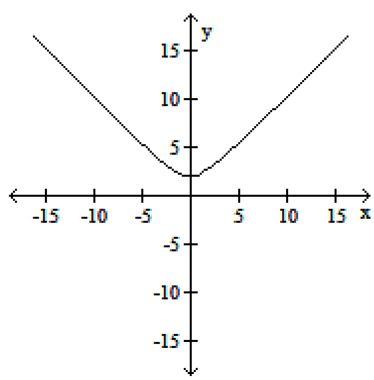
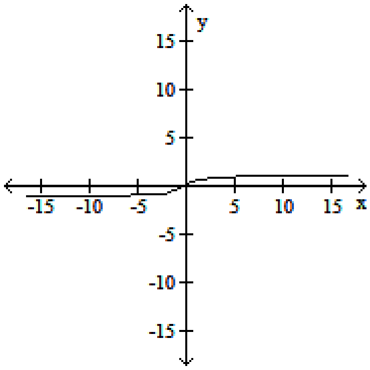


A)



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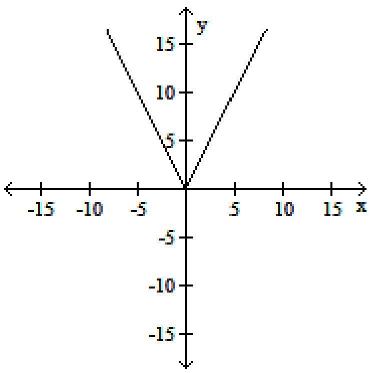

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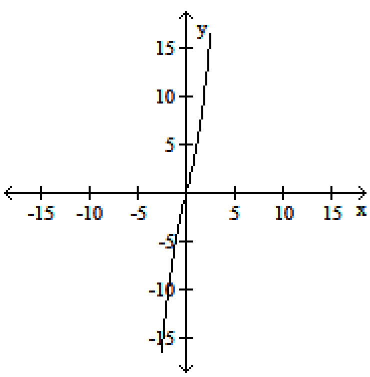
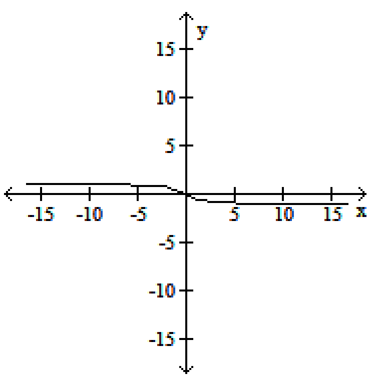


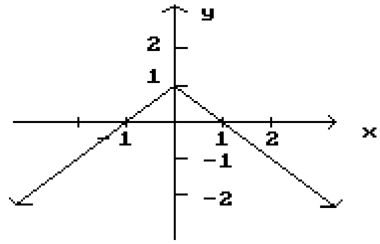
D)



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

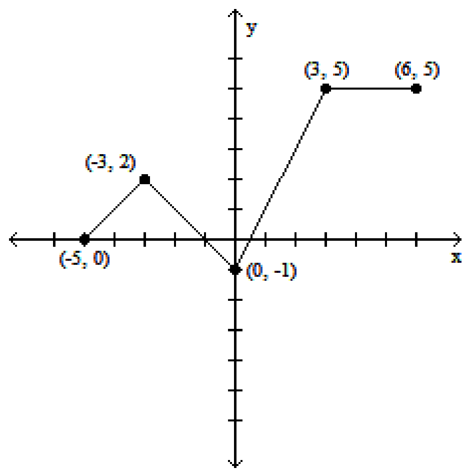
Given the graph of f, find any values of x at which f ′ is not defined.

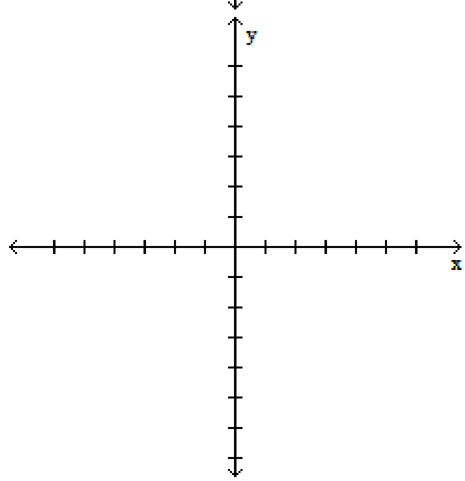
47)



Solve the problem.

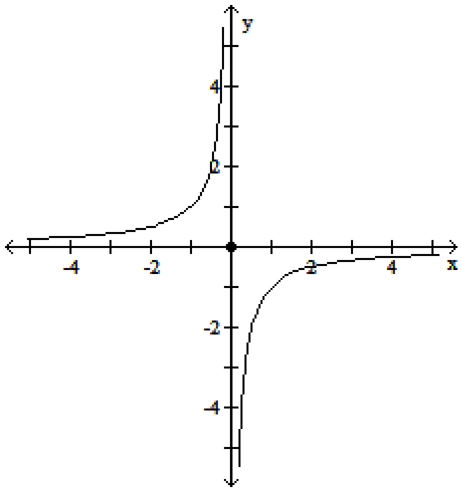
48) The graph of y = f(x) in the accompanying figure is made of line segments joined end to end. Graph the derivative of f.



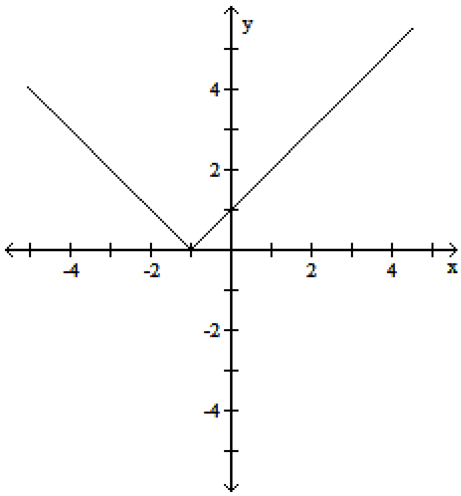


The figure shows the graph of a function. At the given value of x, does the function appear to be differentiable, continuous but not differentiable, or neither continuous nor differentiable?

49) $x = 0$



50) $x = -1$



Find the derivative.

51) $y = 6 - 9x^2$

52) $y = 2x^4 - 9x^3 - 3$

53) $y = 12x^{-2} - 2x^3 + 7x$

Find the second derivative.

54) $y = 7x^4 - 7x^2 + 5$

55) $s = \frac{11t^3}{3} + 11$

56) $y = 3x^2 + 8x + 4x^{-3}$

Find y'.

57) $y = (4x - 2)(3x + 1)$

58) $y = (x^2 - 5x + 2)(3x^3 - x^2 + 5)$

Find the derivative of the function.

59) $y = \frac{x^2 - 3x + 2}{x^7 - 2}$

60) $y = \frac{x^3}{x - 1}$

Suppose u and v are differentiable functions of x. Use the given values of the functions and their derivatives to find the value of the indicated derivative.

61) $u(1) = 2, u'(1) = -6, v(1) = 7, v'(1) = -4.$

$\frac{d}{dx}(uv)$ at $x = 1$

62) $u(1) = 5, u'(1) = -6, v(1) = 7, v'(1) = -4.$

$\frac{d}{dx}\left(\frac{v}{u}\right)$ at $x = 1$

63) $u(1) = 5, u'(1) = -5, v(1) = 6, v'(1) = -4.$

$\frac{d}{dx}(3v - u)$ at $x = 1$

The function $s = f(t)$ gives the position of a body moving on a coordinate line, with s in meters and t in seconds.

64) $s = 6t^2 + 4t + 4, 0 \le t \le 2$

Find the body's displacement and average velocity for the given time interval.

65) $s = -t^3 + 7t^2 - 7t, 0 \le t \le 7$

Find the body's displacement and average velocity for the given time interval.

66) $s = 3t^2 + 4t + 10, 0 \le t \le 2$

Find the body's speed and acceleration at the end of the time interval.

Solve the problem.

67) The position of a body moving on a coordinate line is given by $s = t^2 - 6t + 10$, with s in meters and t in seconds. When, if ever, during the interval $0 \le t \le 6$ does the body change direction?

68) At time t, the position of a body moving along the s-axis is $s = t^3 - 15t^2 + 72t$ m. Find the body's acceleration each time the velocity is zero.

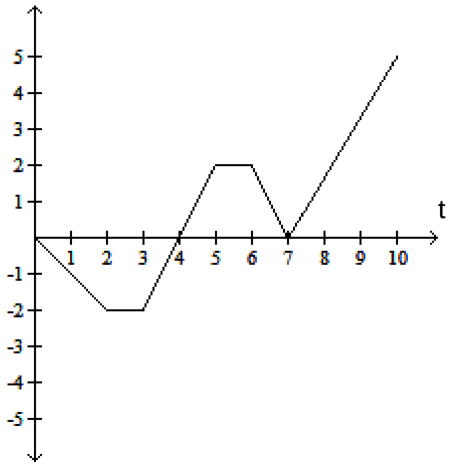
69) At time t, the position of a body moving along the s-axis is $s = t^3 - 18t^2 + 60t$ m. Find the total distance traveled by the body from t = 0 to t = 3.

70) A ball dropped from the top of a building has a height of $s = 400 - 16t^2$ meters after t seconds. How long does it take the ball to reach the ground? What is the ball's velocity at the moment of impact?

71) A rock is thrown vertically upward from the surface of an airless planet. It reaches a height of $s = 120t - 10t^2$ meters in t seconds. How high does the rock go? How long does it take the rock to reach its highest point?

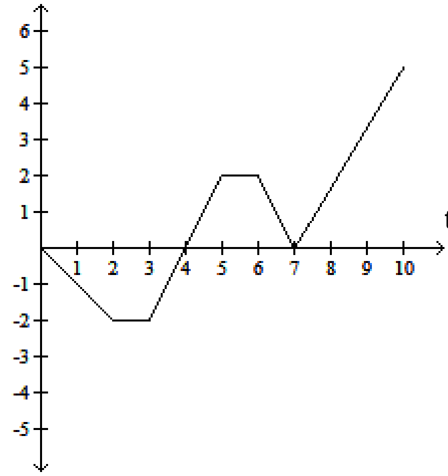
The figure shows the velocity v or position s of a body moving along a coordinate line as a function of time t . Use the figure to answer the question.

72) v (ft/sec)



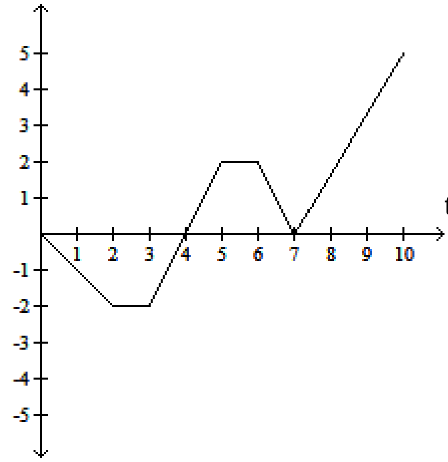
When is the body's acceleration equal to zero?

73) v (ft/sec)



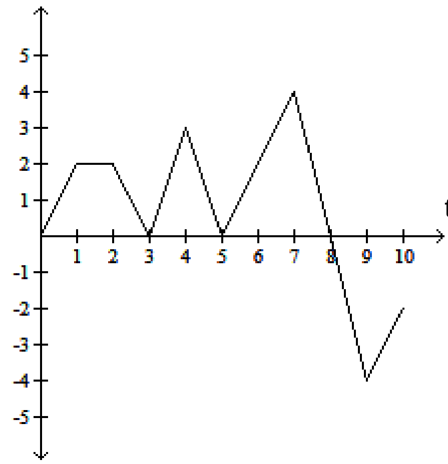
What is the body's acceleration when t = 8 sec?

74) v (ft/sec)



When is the body moving backward?

75) s (m)



When is the body moving forward?

Find the derivative.

76) $y = \frac{9}{x} + 2 \sec x$

77) $s = t^3 \tan t - \sqrt{t}$

78) $y = \frac{6}{\sin x} + \frac{1}{\cot x}$

Find the derivative of the function.

79) $s = \sin\left(\frac{7\pi t}{2}\right) - \cos\left(\frac{7\pi t}{2}\right)$

80) $y = \frac{1}{5}(7x + 10)^3 + \left(1 - \frac{1}{x^3}\right)^{-1}$

81) $y = x^{12/5}$

82) $y = \sqrt[8]{11x}$

Use implicit differentiation to find dy/dx.

83) $2xy - y^2 = 1$

84) $x^3 + 3x^2y + y^3 = 8$

Solve the problem.

85) The line that is normal to the curve $x^2 - xy + y^2 = 9$ at (3, 3) intersects the curve at what other point?

86) Find the normal to the curve $x^2 + y^2 = 2x + 2y$ that is parallel to the line $y + x = 0$.

87) A wheel with radius 2 m rolls at 18 rad/s. How fast is a point on the rim of the wheel rising when the point is $\pi/3$ radians above the horizontal (and rising)? (Round your answer to one decimal place.)

88) Assume that the profit generated by a product is given by $P(x) = 4\sqrt{x}$, where x is the number of units sold. If the profit keeps changing at a rate of $600 per month, then how fast are the sales changing when the number of units sold is 300? (Round your answer to the nearest dollar per month.)

89) A piece of land is shaped like a right triangle. Two people start at the right angle of the triangle at the same time, and walk at the same speed along different legs of the triangle. If the area formed by the positions of the two people and their starting point (the right angle) is changing at 5 m^2/s, then how fast are the people moving when they are 3 m from the right angle? (Round your answer to two decimal places.)

Solve the problem. Round your answer, if appropriate.

90) Water is discharged from a pipeline at a velocity v (in ft/sec) given by $v = 1240p^{(1/2)}$, where p is the pressure (in psi). If the water pressure is changing at a rate of 0.406 psi/sec, find the acceleration (dv/dt) of the water when p = 33.0 psi.

91) Water is being drained from a container which has the shape of an inverted right circular cone. The container has a radius of 9.00 inches at the top and a height of 10.0 inches. At the instant when the water in the container is 6.00 inches deep, the surface level is falling at a rate of 0.9 in./sec. Find the rate at which water is being drained from the container.

92) A man 6 ft tall walks at a rate of 3 ft/sec away from a lamppost that is 21 ft high. At what rate is the length of his shadow changing when he is 30 ft away from the lamppost? (Do not round your answer)

93) The volume of a sphere is increasing at a rate of 8 cm^3/sec. Find the rate of change of its surface area when its volume is $\frac{4\pi}{3}$ cm^3. (Do not round your answer.)

Find the linearization L(x) of f(x) at x = a.

94) $f(x) = 5x^2 - 4x + 1$, a = 3

95) $f(x) = \sqrt{6x + 81}$, a = 0

Find dy.

96) $y = 8x^2 - 8x - 6$

97) $y = \cos(7\sqrt{x})$

Find the absolute extreme values of each function on the interval.

98) $f(x) = 2x - 3; -2 \le x \le 4$

99) $y = -x^2 + 7x - 12$ on [4, 3]

Find the extreme values of the function and where they occur.

100) $y = x^2 + 2x - 3$

101) $y = x^3 - 3x^2 + 1$

102) $y = \dfrac{1}{x^2 - 1}$

103) $y = x^3 - 3x^2 + 5x - 6$

Find the derivative at each critical point and determine the local extreme values.

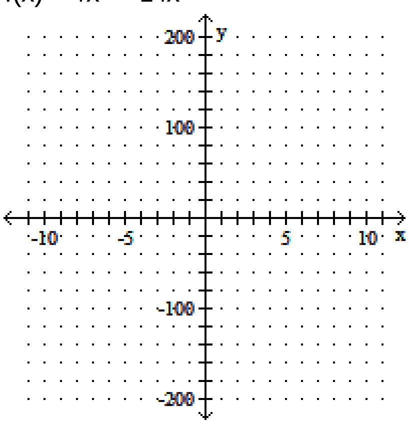
104) $y = x(1 - x^2)$

Give an appropriate answer.

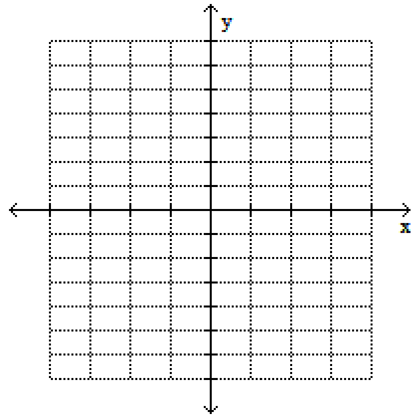
105) Find the value or values of c that satisfy $\dfrac{f(b) - f(a)}{b - a} = f'(c)$ for the function $f(x) = x^2 + 3x + 3$ on the interval [-3, -2].

Sketch the graph and show all local extrema and inflection points.

106) $f(x) = 4x^2 + 24x$



107) $f(x) = 2x^3 - 15x^2 + 24x$



Solve the problem.

108) From a thin piece of cardboard 50 in. by 50 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

109) Find the optimum number of batches (to the nearest whole number) of an item that should be produced annually (in order to minimize cost) if 280,000 units are to be made, it costs \$3 to store a unit for one year, and it costs \$360 to set up the factory to produce each batch.

110) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$R(x) = 20x - 0.5x^2$

$C(x) = 7x + 4.$

111) A rectangular sheet of perimeter 36 cm and dimensions x cm by y cm is to be rolled into a cylinder as shown in part (a) of the figure. What values of x and y give the largest volume?